

Higher Order Corrections to the Primordial Gravitational Wave Spectrum and its Impact on Parameter Estimates for Inflation

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Abstract

We study the impact of the use of the power series expression for the primordial tensor spectrum on parameter estimation from future direct detection gravitational wave experiments. The spectrum approximated by the power series expansion may give large deviation from the true (fiducial) value when it is normalized at CMB scale because of the large separation between CMB and direct detection scales. We derive the coefficients of the higher order terms of the expansion up to the sixth order within the framework of the slow-roll approximation and investigate how well the inclusion of higher order terms improves the analytic prediction of the spectrum amplitude by comparing with numerical results. Using the power series expression, we consider future constraints on inflationary parameters expected from direct detection experiments of the inflationary gravitational wave background and show that the truncation of the higher order terms can lead to incorrect evaluation of the parameters. We present two example models; a quadratic chaotic inflation model and mixed inflaton and curvaton model with a quartic inflaton potential.

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I. INTRODUCTION

Inflation [1] is a successful paradigm not only for solving the horizon and flatness problems, but also for explaining the origin of density perturbations in the Universe. Inflation predicts adiabatic and almost scale-invariant primordial fluctuations, which are in excellent agreement with current observations such as cosmic microwave background (CMB) and so on. However, no direct evidence of inflation has yet been found. During inflation, the gravitational waves could also be produced [2], whose detection can give a direct evidence of inflation and would be a key test of inflation.

Early detection of the inflationary gravitational wave background may be achieved through its unique signature in the polarization of the CMB [3, 4]. The ongoing satellite mission, Planck [5], can detect such indirect signal of gravitational waves if the tensor-to-scalar ratio is $r \gtrsim 0.05$. The next-generation experiment, such as CMBpol [6] and Cosmic Origins Explorer (CORe) [7], are designed to reach $r \sim 10^{-3}$. Moreover, the direct detection may be possible with space-based laser interferometers such as the DECI-hertz Interferometer Gravitational wave Observatory (DECIGO) [8, 9] and Big-Bang Observer (BBO) [10], which would provide independent information about inflation.

While CMB polarization experiments observe large-scale gravitational waves ($k \sim \mathcal{O}(0.001) \text{ Mpc}^{-1}$), space-based laser interferometers measure gravitational waves at millihertz frequencies ($k \sim \mathcal{O}(10^{13}) \text{ Mpc}^{-1}$). This millihertz frequency band is the most prospective region for direct detection of the inflationary gravitational wave background. The detection becomes easier at lower frequencies, since interferometer with longer arms can obtain larger displacement signals by gravitational waves. On the other hand, frequencies below a millihertz would be contaminated by the gravitational wave background generated from white dwarf binaries [11].

We should note that there are also many other mechanisms which may generate a gravitational wave background around the millihertz frequency, such as preheating [12–15], bubble collisions during a first-order phase transition [16], self-ordering scalar fields following a global phase transition [20, 21], second order effects from enhanced scalar perturbations [22–25], topological defects [26–32], supernova explosions of population III first stars [33–35], gamma-ray bursts [36], and so on. However, their amplitude and frequency strongly depend on their unknown physics, so the millihertz band is still a window to search for the

inflationary gravitational wave background. In this paper, we focus on the gravitational wave background from inflation and do not consider other sources which may contaminate the millihertz band.

The large difference between CMB and direct detection scales means that these two types of observations enable us to look at different periods of inflation, which would greatly help to investigate the inflaton potential [37–43]. However, we should carefully choose the method to connect the two separate scales. A common method is to use a power-law extrapolation from CMB scales to direct detection scales for describing the primordial tensor spectrum. Yet recent works [44–46] have pointed out that the Taylor expansion around the CMB scale is no longer valid at the direct detection frequency and it causes an incorrect estimation of the amplitude of the inflationary gravitational wave background.

One way to avoid the wrong estimation of the spectrum is to resort to a full numerical calculation to obtain the gravitational wave spectrum. However, the power-law extrapolation is much simpler and easier than the numerical method and, in principle, its precision can be improved by including higher order terms in the Taylor expansion as much as possible. In this paper, we derive the slow-roll expression for the primordial tensor power spectrum up to the sixth order in the Taylor expansion and examine how much the inclusion of the higher order terms improves the estimation of the amplitude at direct detection scales by comparing with the full numerical computation [46]. Furthermore, we discuss the impact of the truncation of the higher order terms in the power series expansion of the tensor spectrum by presenting constraints on inflationary parameters expected from future direct detection experiments, which is an example that such a poor estimation of the spectrum amplitude causes a problem.

This paper is organized as follows: In Sec. II, we give a formula of the power series expression for the primordial tensor spectrum including up to the sixth order in the Taylor expansion. Next, in Sec. III, we discuss whether the expression given in Sec. II well describes the tensor power spectrum by comparing those with numerically obtained spectra. We consider two example models for the comparison, the chaotic inflation model with quadratic and quartic potentials. Although the quartic chaotic inflation is already excluded by observations such as CMB, by adding the contribution from another source of fluctuations such as the curvaton, the quartic inflation model can be allowed due to the existence of the curvaton fluctuations, which is sometimes called mixed inflaton and curvaton model

[47–50]. Note that this kind of mixed scenario can give sizable tensor-to-scalar ratio as well as large non-Gaussianity, which might be interesting from the viewpoint of near future observations. In Sec. IV, we give expected constraints on the inflationary parameters for the above mentioned two models. In passing, we discuss to what extent the truncation of the tensor spectrum expression at some (lower) order leads to incorrect evaluation of the inflationary parameters. Finally, we conclude in Sec. V.

II. SLOW-ROLL FORMALISM AND POWER SERIES EXPANSION

In the standard picture of the early universe, a scalar field ϕ , the inflaton, drives superluminal cosmic expansion, the inflation. The equation of motion for ϕ is given by

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0, \quad (1)$$

where the dot and prime denote the derivative with respect to t and ϕ , respectively. The dynamics of inflation is often characterized by the slow-roll parameters. In this paper, we work with the slow-roll parameters which are defined in terms of the inflaton potential V and its derivatives as [51]

$$\begin{aligned} \epsilon_V &\equiv \frac{M_{\text{Pl}}^2}{2} \left(\frac{V'}{V} \right)^2, \\ \eta_V &\equiv M_{\text{Pl}}^2 \frac{V''}{V}, \\ \xi_V^2 &\equiv M_{\text{Pl}}^4 \frac{V'V'''}{V^2}, \\ \sigma_V^3 &\equiv M_{\text{Pl}}^6 \frac{V'^2V^{(4)}}{V^3}, \\ \tau_V^4 &\equiv M_{\text{Pl}}^8 \frac{V'^3V^{(5)}}{V^4}, \\ \zeta_V^5 &\equiv M_{\text{Pl}}^{10} \frac{V'^4V^{(6)}}{V^5}, \end{aligned} \quad (2)$$

where the subscript (n) denotes the n -th derivative with respect to ϕ . Inflation lasts as long as $\epsilon_V, |\eta_V| \ll 1$, which are called the slow-roll conditions, and it ends when this condition is violated, $\max\{\epsilon_V(\phi_{\text{end}}), \eta_V(\phi_{\text{end}})\} = 1$. In the slow-roll limit, the evolution of the Hubble parameter $H(t)$ is given by $H^2 \simeq V/(3M_{\text{Pl}}^2)$, where $M_{\text{Pl}} = 1/\sqrt{8\pi G}$ is the reduced Planck mass. The duration of inflation is characterized by the e-folding number, $N \equiv \ln(a_{\text{end}}/a)$,

which can be rewritten in terms of the potential,

$$N \simeq \frac{1}{M_{\text{Pl}}^2} \int_{\phi_{\text{end}}}^{\phi} \frac{V}{V'} d\phi. \quad (3)$$

Within the slow-roll approximation, the primordial power spectra of scalar and tensor perturbations are given by [52, 53]

$$\mathcal{P}_S \simeq [1 - (2C + 1)\epsilon_H + C\eta_H]^2 \frac{1}{16\pi^2 M_{\text{Pl}}^4} \frac{H^4}{H'^2} \Big|_{k=aH}, \quad (4)$$

$$\mathcal{P}_T \simeq [1 - (C + 1)\epsilon_H]^2 \frac{2}{\pi^2 M_{\text{Pl}}^2} H^2 \Big|_{k=aH}, \quad (5)$$

where $C = -2 + \ln 2 + \gamma \simeq -0.73$ with γ being the Euler constant and ϵ_H and η_H are the Hubble slow-roll parameters, $\epsilon_H \equiv 2M_{\text{Pl}}^2(H'/H)^2$ and $\eta_H \equiv 2M_{\text{Pl}}^2 H''/H$. Hereafter, we only consider the leading order for the slow-roll parameters. Then the power spectra are given in terms of the inflaton potential as

$$\mathcal{P}_S \simeq \frac{1}{12\pi^2 M_{\text{Pl}}^6} \frac{V^3}{V'^2} \Big|_{k=aH}, \quad (6)$$

$$\mathcal{P}_T \simeq \frac{2}{3\pi^2 M_{\text{Pl}}^4} V \Big|_{k=aH}. \quad (7)$$

They are evaluated at the moment when each Fourier mode k crosses the Hubble horizon, as indicated by the subscript “ $k = aH$.” It is often assumed that the values of V and its derivatives evolve so slowly during inflation that the spectra can be parametrized by using the Taylor expansion in terms of the logarithm of the wave number,

$$\begin{aligned} \mathcal{P}_T(k) = \mathcal{P}_{T\star} \exp & \left[n_{T\star} \ln \frac{k}{k_\star} + \frac{1}{2!} \alpha_{T\star} \ln^2 \frac{k}{k_\star} + \frac{1}{3!} \beta_{T\star} \ln^3 \frac{k}{k_\star} \right. \\ & \left. + \frac{1}{4!} \gamma_{T\star} \ln^4 \frac{k}{k_\star} + \frac{1}{5!} \delta_{T\star} \ln^5 \frac{k}{k_\star} + \frac{1}{6!} \theta_{T\star} \ln^6 \frac{k}{k_\star} + \dots \right], \end{aligned} \quad (8)$$

where the coefficients are the parameters characterizing a deviation from the scale-invariant

spectrum,

$$\begin{aligned}
n_T(k) &\equiv \frac{d \ln \mathcal{P}_T(k)}{d \ln k}, \\
\alpha_T(k) &\equiv \frac{dn_T(k)}{d \ln k}, \\
\beta_T(k) &\equiv \frac{d\alpha_T(k)}{d \ln k}, \\
\gamma_T(k) &\equiv \frac{d\beta_T(k)}{d \ln k}, \\
\delta_T(k) &\equiv \frac{d\gamma_T(k)}{d \ln k}, \\
\theta_T(k) &\equiv \frac{d\delta_T(k)}{d \ln k}.
\end{aligned} \tag{9}$$

The expression for the scalar power spectrum is the same except that the coefficient of the first term is $(n_{S\star} - 1)$. The subscript \star denotes quantities evaluated at the pivot scale, which is commonly taken to be the scale of the CMB, $k_\star = 0.002 \text{Mpc}^{-1}$. The coefficients can be given in terms of the slow-roll parameters as

$$\begin{aligned}
n_T(k) &\simeq -2\epsilon_V \\
\alpha_T(k) &\simeq -4\epsilon_V[2\epsilon_V - \eta_V], \\
\beta_T(k) &\simeq -4\epsilon_V[16\epsilon_V^2 + 2\eta_V^2 - 14\epsilon_V\eta_V + \xi_V^2], \\
\gamma_T(k) &\simeq -4\epsilon_V[192\epsilon_V^3 - 236\epsilon_V^2\eta_V + 72\epsilon_V\eta_V^2 - 4\eta_V^3 + 22\epsilon_V\xi_V^2 - 7\eta_V\xi_V^2 - \sigma_V^3], \\
\delta_T(k) &\simeq -4\epsilon_V[3042\epsilon_V^4 - 4810\epsilon_V^3\eta_V + 2280\epsilon_V^2\eta_V^2 - 328\epsilon_V\eta_V^3 + 8\eta_V^4 \\
&\quad + 500\epsilon_V^2\xi_V^2 - 324\epsilon_V\eta_V\xi_V^2 + 33\eta_V^2\xi_V^2 + 7\xi_V^4 - 32\epsilon_V\sigma_V^3 + 11\eta_V\sigma_V^3 + \tau_V^4], \\
\theta_T(k) &\simeq -4\epsilon_V[61440\epsilon_V^5 - 117840\epsilon_V^4\eta_V + 75200\epsilon_V^3\eta_V^2 - 18272\epsilon_V^2\eta_V^3 + 1408\epsilon_V\eta_V^4 - 16\eta_V^5 \\
&\quad + 12840\epsilon_V^3\xi_V^2 - 12596\epsilon_V^2\eta_V\xi_V^2 + 3000\epsilon_V\eta_V^2\xi_V^2 - 131\eta_V^3\xi_V^2 + 408\epsilon_V\xi_V^4 - 94\eta_V\xi_V^4 \\
&\quad - 948\epsilon_V^2\sigma_V^3 + 648\epsilon_V\eta_V\sigma_V^3 - 77\eta_V^2\sigma_V^3 - 25\xi_V^2\sigma_V^3 + 44\epsilon_V\tau_V^4 - 16\eta_V\tau_V^4 - \zeta_V^5].
\end{aligned} \tag{10}$$

The amplitude of the tensor perturbation at the CMB scale is often parametrized by the tensor-to-scalar ratio:

$$r \equiv \frac{\mathcal{P}_{T\star}}{\mathcal{P}_{S\star}} = 16\epsilon_{V\star}. \tag{11}$$

As we will show in the next section, the inclusion of the higher order terms in the Taylor expansion up to the 6th order seems to be in very good agreement with a full numerical calculation, which indicates that the above expressions would be precise enough to give correct tensor power spectra for many inflation models and can be used for parameter estimation from observational data.

III. OVERESTIMATION OF THE TENSOR POWER SPECTRUM

In most works, it is common to simply adopt the power-law extrapolation from CMB scales to direct detection scales for describing the gravitational wave background spectrum. However, as we will show below, such a power-law extrapolation may not be valid and lead to an incorrect estimation of the spectrum amplitude at direct detection scales. In Figure 1, we show the gravitational wave spectra calculated using the Taylor expansion truncating at some order and the one obtained from full numerical computations. Here we consider quadratic (ϕ^2) and quartic (ϕ^4) chaotic inflation models. For the ϕ^4 model, we consider a mixed inflaton and curvaton model where fluctuations from the curvaton [54–56] also contribute to cosmic density perturbations. This is because the quartic chaotic inflation model predicts too large tensor-to-scalar ratio which is already excluded by current observational data. In addition, the curvaton model can generate large non-Gaussianity, thus such a mixed model would be interesting to investigate since it can produce both sizable gravitational wave amplitude and large non-Gaussianity detectable in the near future observations ¹.

As seen from the figure, the power series expression overestimates the amplitude of the spectrum because of the large separation between the two scales. The spectra are plotted using Eq. (8) by truncating the Taylor expansion at each order, respectively. The exact spectrum, which is obtained from a numerical calculation [46], is also plotted for comparison. The truncation of the higher order terms in Eq. (8) is the cause of the overestimation because the contribution of the higher order terms is non-negligible as they are boosted by the n -th power of $\ln(k_{0.2\text{Hz}}/k_\star) \simeq 38.7$, even though the coefficients of the n -th terms are suppressed as ϵ^n . The overestimation of the spectrum amplitude can be avoided if the slow-roll parameters are much smaller than $[\ln(k_{0.2\text{Hz}}/k_\star)]^{-1} \simeq 2.58 \times 10^{-2}$, but this is not the case, in particular, for chaotic inflation models.

Table I lists the values of \mathcal{P}_T at direct detection frequency ($f = 0.2\text{Hz}$) for cases of the truncation at each order in the Taylor expansion. The degree of overestimation compared to the numerical result is presented in percentage. We also list the values converted to the density parameter of the gravitational wave background, $\Omega_{\text{GW}} \equiv (d\rho_{\text{GW}}/d\ln k)/\rho_{c,0}$ [57], where $\rho_{c,0} \equiv 3M_{\text{Pl}}^2 H_0^2$ is the critical density of the Universe today and ρ_{GW} is the energy

¹ Mixed inflaton-curvaton models have been studied in [47–50] and we refer the readers to these papers for details.

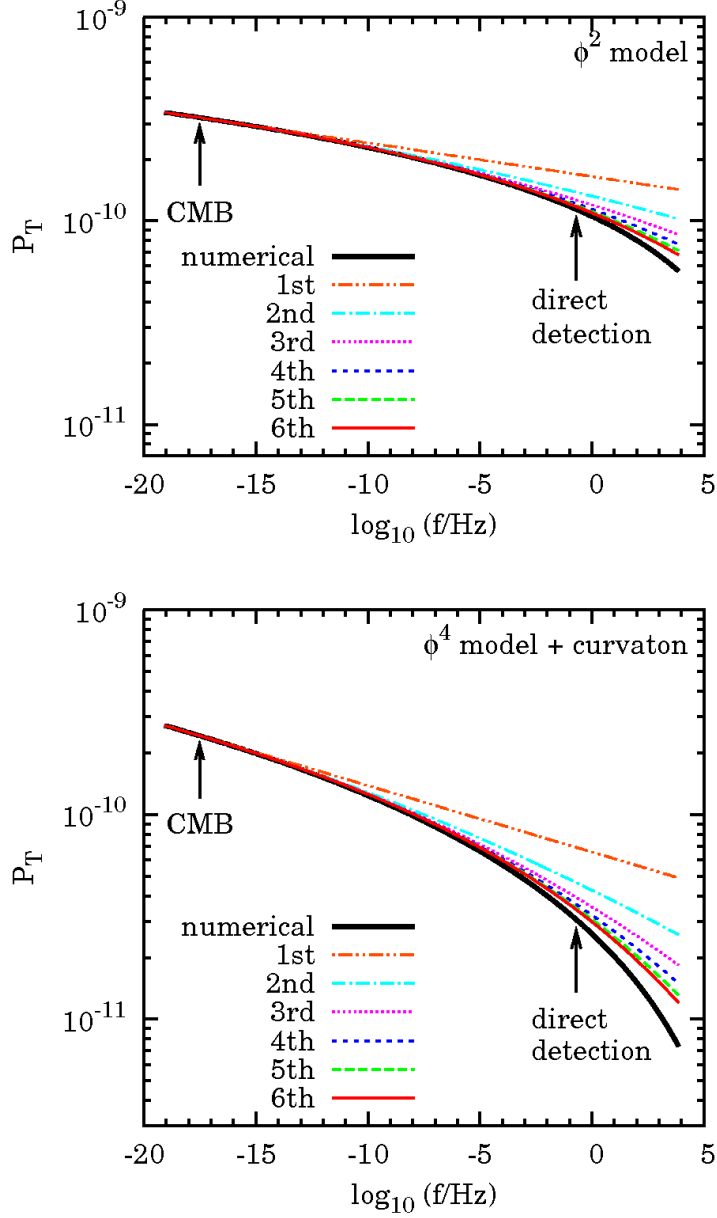


FIG. 1: Comparison between the exact (numerically obtained) spectrum and the spectra approximated by truncating the Taylor expansion after the first, second, third, fourth, fifth or sixth order terms in Eq. (8). The primordial tensor spectra \mathcal{P}_T are plotted against frequency, $f = k/2\pi$. The upper panel shows the case with the quadratic chaotic inflation model, $V = m^2\phi^2/2$. The bottom panel shows the case for a mixed inflaton and curvaton model with the quartic inflaton potential, $V = \lambda\phi^4/4$. The fraction of the curvaton contribution is fixed requiring that the tensor-to-scalar ratio is $r = 0.1$.

density of the gravitational waves. The primordial tensor spectrum $\mathcal{P}_T(k)$ can be converted to the present-day density parameter by using the transfer function as

$$\Omega_{\text{GW}} = \frac{1}{12} \left(\frac{k}{H_0} \right)^2 \mathcal{P}_T(k) T_T^2(k). \quad (12)$$

The transfer function is given by

$$T_T^2(k) = (1 - \Omega_\Lambda)^2 \left(\frac{g_*(T_{\text{hc}})}{g_{*0}} \right) \left(\frac{g_{*s0}}{g_{*s}(T_{\text{hc}})} \right)^{4/3} \left(\frac{3}{\sqrt{2}(k\tau_0)^2} \right)^2 (1 + 1.57x_{\text{eq}} + 3.42x_{\text{eq}}^2), \quad (13)$$

where $\tau_0 = 2H_0^{-1}$, $x_{\text{eq}} = k/k_{\text{eq}}$ and $k_{\text{eq}} \equiv \tau_{\text{eq}}^{-1} = 7.1 \times 10^{-2} \Omega_m h^2 \text{Mpc}^{-1}$ [41, 58]. The effective number of degrees of freedom is given as $g_*(T_{\text{hc}}) = g_{*s}(T_{\text{hc}}) = 106.75$, when the contribution from the relativistic standard model particles are taken into account. The values at present are $g_{*0} = 3.36$ and $g_{*s0} = 3.90$. If we assume the cosmological parameters to be $\Omega_\Lambda = 0.734$, $\Omega_m h^2 = 0.1334$, $h = 0.710$ (taken from the WMAP 7-year mean values [59]), the amplitude of the primordial spectrum at the direct detection frequency $f = 0.2 \text{ Hz}$ is given by

$$\Omega_{\text{GW},0.2\text{Hz}} = 1.36 \times 10^{-6} \mathcal{P}_{T,0.2\text{Hz}}. \quad (14)$$

Figure 2 plots the amplitude of the present-day tensor spectrum at direct detection scale ($f = 0.2\text{Hz}$) in terms of Ω_{GW} for different order truncation for the above mentioned two models. From the figure, we see that the inclusion of the higher order terms improves the overestimation significantly.

In figure 3, we show how much the overestimation of the amplitude affects determination of n_T . One may try to determine the tilt of the spectrum n_T if the amplitude of the gravitational wave background is determined by both CMB and direct detection experiments. However, truncation of the higher order terms would yield wrong value of n_T . The values listed in table I and plotted in figure 3 are estimated with Eq. (8) truncating at each order, with the assumption that \mathcal{P}_T is determined at both the CMB k_* and direct detection scales $k_{0.2\text{Hz}}$.

Below we give some detailed discussion for models considered here: the quadratic chaotic inflation and a mixed inflaton and curvaton model with a quartic inflaton potential.

A. ϕ^2 model

In the case of the chaotic inflation with a quadratic potential,

$$V = \frac{1}{2} m^2 \phi^2, \quad (15)$$

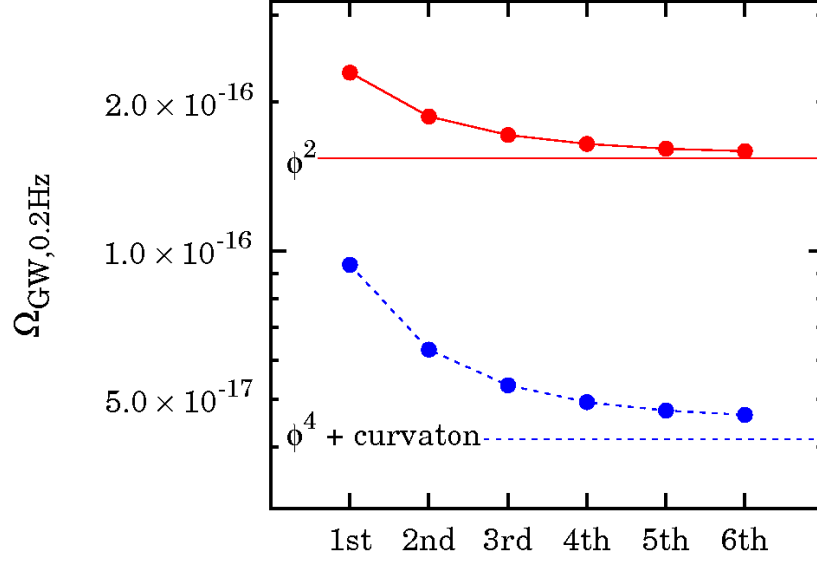


FIG. 2: Comparison of the overestimation for different order truncation. The vertical axis shows the amplitude of the gravitational wave background spectrum Ω_{GW} at the direct detection frequency $f = 0.2\text{Hz}$. The points represent values calculated by Eq. (8) truncated at each order. The straight lines correspond to the exact values obtained from the numerical calculation.

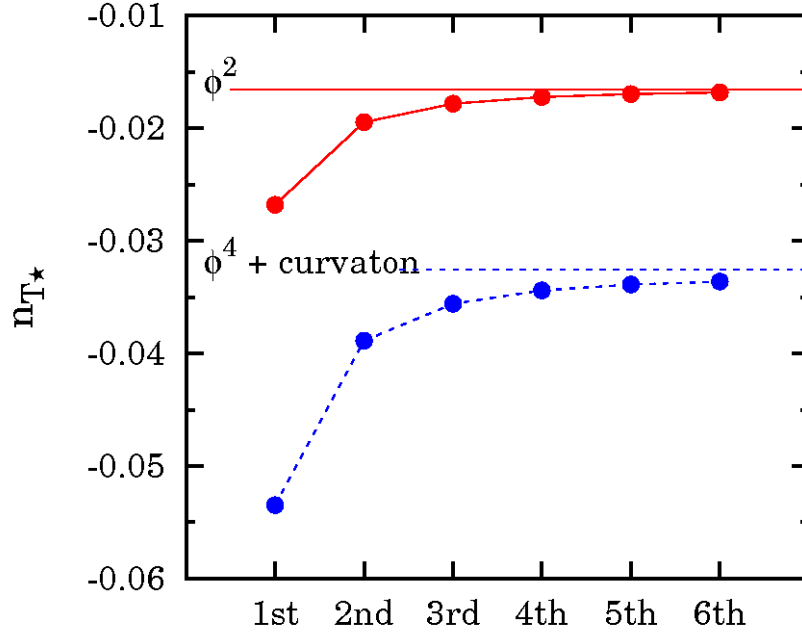


FIG. 3: Comparison of the values of n_T estimated with Eq. (8) truncated at different order.

	\mathcal{P}_T	Ω_{GW}	overestimation (%)	$n_{T\star}$
$\phi^2\text{model}$				
numerical	1.14×10^{-10}	1.54×10^{-16}	0	-0.0165
1st	1.69×10^{-10}	2.30×10^{-16}	49	-0.0268
2nd	1.38×10^{-10}	1.87×10^{-16}	21	-0.0195
3rd	1.27×10^{-10}	1.72×10^{-16}	11	-0.0178
4th	1.21×10^{-10}	1.64×10^{-16}	7	-0.0172
5th	1.19×10^{-10}	1.61×10^{-16}	4	-0.0170
6th	1.17×10^{-10}	1.59×10^{-16}	3	-0.0168
$\phi^4\text{model} + \text{curvaton } (r = 0.1)$				
numerical	3.07×10^{-11}	4.15×10^{-17}	0	-0.0325
1st	6.90×10^{-11}	9.35×10^{-17}	125	-0.0535
2nd	4.64×10^{-11}	6.29×10^{-17}	51	-0.0389
3rd	3.93×10^{-11}	5.33×10^{-17}	28	-0.0356
4th	3.64×10^{-11}	4.93×10^{-17}	19	-0.0344
5th	3.49×10^{-11}	4.74×10^{-17}	14	-0.0339
6th	3.42×10^{-11}	4.64×10^{-17}	12	-0.0336

TABLE I: Summary of the amplitude of the primordial tensor spectrum \mathcal{P}_T , the present-day density parameter of gravitational wave background Ω_{GW} and percentage of overestimation due to the Taylor expansion, which are all evaluated at the direct detection frequency $f = 0.2\text{Hz}$. The values of $n_{T\star}$ evaluated with the truncated expression of the spectrum are also listed.

the slow-roll parameters are given as

$$\begin{aligned}\epsilon_V &= \eta_V = 2 \frac{M_{\text{Pl}}^2}{\phi^2}, \\ \xi_V^2 &= \sigma_V^3 = \tau_V^4 = \zeta_V^5 = 0,\end{aligned}\tag{16}$$

and Eq. (3) gives

$$N = \frac{\phi^2}{4M_{\text{Pl}}^2} - \frac{1}{2}.\tag{17}$$

From Eq. (6) we obtain

$$\mathcal{P}_S \simeq \frac{1}{96\pi^2 M_{\text{Pl}}^6} m^2 \phi^4,\tag{18}$$

and from Eq. (7)

$$\mathcal{P}_T \simeq \frac{1}{3\pi^2 M_{\text{Pl}}^4} m^2 \phi^2. \quad (19)$$

The spectrum is calculated assuming that the e-folding number corresponding to the CMB scale is $N_\star = \ln(a_{\text{end}}/a_\star) = 60$, which gives $\epsilon_{V\star} = 8.26 \times 10^{-3}$ and $n_{T\star} \simeq -1.65 \times 10^{-2}$, $\alpha_{T\star} \simeq -2.73 \times 10^{-4}$, $\beta_{T\star} \simeq -9.03 \times 10^{-6}$, $\gamma_{T\star} \simeq -4.48 \times 10^{-7}$, $\delta_T \simeq -2.96 \times 10^{-8}$, $\theta_T \simeq -2.45 \times 10^{-9}$. The mass of the inflaton field $m = 1.53 \times 10^{13} \text{GeV}$ is determined to satisfy the normalization of the scalar perturbations, $\mathcal{P}_{S\star} = 2.43 \times 10^{-9}$, which gives $\mathcal{P}_{T\star} = 3.21 \times 10^{-10}$. In this model, the tensor-to-scalar ratio and the scalar spectral index are $r = 0.132$ and $n_s = 0.967$, respectively.

B. ϕ^4 model with the curvaton

If we consider the chaotic inflation model with a quartic potential,

$$V = \frac{1}{4} \lambda \phi^4, \quad (20)$$

the slow-roll parameters are given as

$$\begin{aligned} \epsilon_V &= 8 \frac{M_{\text{Pl}}^2}{\phi^2}, \\ \eta_V &= 12 \frac{M_{\text{Pl}}^2}{\phi^2} = \frac{3}{2} \epsilon_V, \\ \xi_V^2 &= 96 \frac{M_{\text{Pl}}^4}{\phi^4} = \frac{3}{2} \epsilon_V^2, \\ \sigma_V^3 &= 384 \frac{M_{\text{Pl}}^6}{\phi^6} = \frac{3}{4} \epsilon_V^3, \\ \tau_V^4 &= \zeta_V^5 = 0, \end{aligned} \quad (21)$$

and Eq. (3) gives

$$N = \frac{\phi^2}{8M_{\text{Pl}}^2} - \frac{3}{2}. \quad (22)$$

We again take the value of the e-folding number as $N_\star = 60$, which gives $\epsilon_{V\star} = 1.63 \times 10^{-2}$ and $r = 0.26$. This large tensor-to-scalar ratio is already excluded by current observational constraints, but it can be avoided by introducing the curvaton fluctuations.

In the curvaton scenario, the fluctuations in the curvaton field σ produce the scalar perturbations, which results in a different expression for the tensor-to-scalar ratio [47–50]. Since our interest is in the case where the detection of the gravitational wave background is

possible, we assume here that the tensor-to-scalar ratio is $r = 0.1$. In this case, fluctuations both from the inflaton and the curvaton contribute to the primordial curvature perturbation. In this model, the scalar power spectrum is given by

$$\mathcal{P}_S = \mathcal{P}_S^{(\phi)} + \mathcal{P}_S^{(\sigma)} = (1 + \alpha)\mathcal{P}_S^{(\phi)}, \quad (23)$$

where $\mathcal{P}_S^{(\phi)}$ and $\mathcal{P}_S^{(\sigma)}$ are the contributions from the inflaton and the curvaton, respectively. α represents the ratio of the curvaton power spectrum to the inflaton one at the reference scale, i.e., $\alpha = \mathcal{P}_S^{(\sigma)} / \mathcal{P}_S^{(\phi)}$. For the ϕ^4 potential, $\mathcal{P}_S^{(\phi)}$ is given by

$$\mathcal{P}_S^{(\phi)} \simeq \frac{1}{768\pi^2 M_{\text{Pl}}^6} \lambda \phi^6. \quad (24)$$

The scalar spectral index is also modified as

$$n_S = 1 - 2\epsilon_V - \frac{4\epsilon_V - 2\eta_V}{1 + \alpha}. \quad (25)$$

Although the expressions for the scalar perturbation quantities are modified in this kind of mixed models, the formulae for the tensor perturbation spectrum \mathcal{P}_T and the parameters for its scale dependence, n_T , α_T , β_T , γ_T , δ_T , θ_T , are not modified from the usual inflationary predictions without the curvaton. The tensor spectrum in the ϕ^4 chaotic inflation model is obtained from Eq. (7) as

$$\mathcal{P}_T \simeq \frac{1}{6\pi^2 M_{\text{Pl}}^4} \lambda \phi^4. \quad (26)$$

Since the scalar power spectrum is modified as in Eq. (23), the tensor-to-scalar ratio is given by

$$r = \frac{16\epsilon_V}{1 + \alpha}. \quad (27)$$

Assuming $N_\star = 60$, we obtain $n_{T\star} \simeq -3.25 \times 10^{-2}$, $\alpha_{T\star} \simeq -5.29 \times 10^{-4}$, $\beta_{T\star} \simeq -1.72 \times 10^{-5}$, $\gamma_{T\star} \simeq -8.39 \times 10^{-7}$, $\delta_T \simeq -5.46 \times 10^{-8}$ and $\theta_T \simeq -4.43 \times 10^{-9}$. Given the value $\epsilon_{V\star} = 1.63 \times 10^{-2}$, our assumption of $r = 0.1$ corresponds to $\alpha_\star = 1.6$. The normalization of the scalar perturbations, $\mathcal{P}_{S\star} = 2.43 \times 10^{-9}$, is used to determine $\lambda = 5.94 \times 10^{-14}$, which gives $\mathcal{P}_{T\star} = 2.43 \times 10^{-10}$. With this setup, the spectral index for the scalar perturbation is $n_s = 0.961$.

Here we briefly comment on non-Gaussianity in this scenario. Usually non-Gaussianity of density fluctuations is represented by so-called non-linearity parameter f_{NL} , which charac-

terizes the size of 3-point function or bispectrum ². Since the standard single-field inflation model predicts very small values of f_{NL} as $f_{\text{NL}} \ll \mathcal{O}(1)$, if the values of f_{NL} is found to be large in the future, it indicates that we need another source of density fluctuations other than the inflaton. As another mechanism of density fluctuations, the curvaton model [54–56] has been intensively investigated, and in particular, this model can generate large non-Gaussianity. Even if fluctuations from the inflaton also contribute to the density fluctuations in the Universe, as far as the curvaton also generates some fraction of the fluctuations, $f_{\text{NL}}^{\text{local}}$ can be large. Furthermore, large tensor-to-scalar ratio is also possible in this model, which can be detectable in the near future. Note that, when the curvaton is the only source of density fluctuations, which is usually assumed in many works, the tensor-to-scalar ratio becomes very small. However, this kind of mixed model can give sizable f_{NL} and r .

In this mixed scenario where local-type non-Gaussianity is generated, f_{NL} is given by [50, 60]

$$f_{\text{NL}} = \left(\frac{\alpha}{1 + \alpha} \right)^2 f_{\text{NL}}^{(\text{curvaton})}. \quad (28)$$

Here $f_{\text{NL}}^{(\text{curvaton})}$ is the one for pure curvaton model (the curvaton is the only source of density fluctuation). Depending on the mass, the decay rate and the initial amplitude of the curvaton field, $f_{\text{NL}}^{(\text{curvaton})}$ can be very large. Thus, by tuning these parameters, the case of $\alpha_\star = 1.6$ (and $r = 0.1$), which is assumed in this section, can also give large f_{NL} . Hence, once the gravitational waves and (large) non-Gaussianity are detected, this kind of scenario would be worth investigating carefully [61].

IV. IMPACT ON PARAMETER ESTIMATION

Now, in this section, we study the influence of the poor estimation of the gravitational spectrum amplitude when one adopts the Taylor approximation truncated at some order. If direct detection determines the amplitude of the inflationary gravitational wave background, one may try to extract information on the inflaton potential and the e-folding number [41] by combining observations of CMB [62–64] and other complementary experiments [65, 66]. However, as shown in the previous section, when the power series expression of the spectrum

² Current constraints on local- equilateral- and orthogonal-types of f_{NL} are (95 % C.L.) [59]: $-10 < f_{\text{NL}}^{\text{local}} < 74$, $-214 < f_{\text{NL}}^{\text{equil}} < 266$ and $-410 < f_{\text{NL}}^{\text{equil}} < 6$, respectively.

is adopted, one would overestimate the amplitude of the gravitational wave spectrum at the direct detection scale if one truncates the expression at some lower order. Here we present how such overestimation of the amplitude affects the determination of the inflationary parameters by investigating future constraints. In this section, we again consider the models discussed in the previous section. Note that, in this paper, we do not consider effect of reheating which may change the shape of the inflationary gravitational wave background around the direct detection frequency [67–69].

A. ϕ^2 model

If the quadratic chaotic inflation is the model realized in the nature, the inflationary gravitational wave background could be directly detected with $\Omega_{\text{GW},0.2\text{Hz}} = 1.54 \times 10^{-16}$ (taken from the numerical result, given in Table I), which is obtained assuming $N_\star = 60$ and the scalar perturbation being normalized as $\mathcal{P}_{S\star} = 2.43 \times 10^{-9}$. With the power-law approximation, one can describe the amplitude of the gravitational wave background at direct detection scale as

$$\Omega_{\text{GW},0.2\text{Hz}} = 1.36 \times 10^{-6} \mathcal{P}_{T\star} \exp[-2(38.7\epsilon_{V\star}) - \frac{4}{2!}(38.7\epsilon_{V\star})^2 - \frac{16}{3!}(38.7\epsilon_{V\star})^3 - \frac{96}{4!}(38.7\epsilon_{V\star})^4 - \frac{768}{5!}(38.7\epsilon_{V\star})^5 - \frac{7680}{6!}(38.7\epsilon_{V\star})^6 + \dots], \quad (29)$$

where we have used Eqs. (8), (10), (14), (16) and $\ln(k_{0.2\text{Hz}}/k_\star) = 38.7$. Notice that, from the above expression, the relation between $\mathcal{P}_{T\star}$ and $\epsilon_{V\star}$ can be provided once the value of $\Omega_{\text{GW},0.2\text{Hz}}$ is determined. The values of $\mathcal{P}_{T\star}$ and $\epsilon_{V\star}$ directly give information on the e-folding number N_\star and the mass of the inflaton m via the following relations,

$$N_\star = \frac{1}{2\epsilon_{V\star}} - \frac{1}{2}, \quad (30)$$

$$m^2 = \frac{3\pi^2 M_{\text{Pl}}^2 \epsilon_{V\star}}{2} \mathcal{P}_{T\star}, \quad (31)$$

which follow from Eqs. (17) and (19).

In Figure 4, we show parameter constraints expected from future CMB observations in the $m - N_\star$ plane as well as the values of m and N_\star which give $\Omega_{\text{GW},0.2\text{Hz}} = 1.54 \times 10^{-16}$ at the direct detection scale for several cases of the truncation in Taylor expansion at some order. Expected CMB constraints are derived from the Fisher matrix analysis [3, 4] with

the instrumental sensitivity of Planck [5] and CMBpol [6], taking into account the analysis of both temperature and polarization data up to the multipole $l = 2000$. The uncertainties on m and N_\star are obtained by transforming parameters from $(n_s, r, \mathcal{P}_{S\star})$ into (m, N_\star) [41], with other cosmological parameters $(h, \Omega_b h^2, \Omega_c h^2, \tau) = (0.710, 0.1109, 0.02258, 0.088)$ [59] marginalized over.

Figure 4 illustrates an important fact that the values of m and N_\star are estimated incorrectly when one determines the parameters from direct detection experiments using the power series expression with higher order terms being neglected. The lines in the $m - N_\star$ plane are plotted by Eqs. (30) and (31) with parameters $\mathcal{P}_{T\star}$ and $\epsilon_{V\star}$ satisfying Eq. (29), truncated at each order. The fiducial values of m and N_\star are taken to be the same as in Sec. III. Neglect of the higher order terms leads to an underestimation of $\mathcal{P}_{T\star}$ or an overestimation of $\epsilon_{V\star}$, which results in an incorrect estimation of the values of m and N_\star . As seen from the figure, the deviation of the line from the true (fiducial) value becomes larger as the power series expansion is truncated at lower order.

In particular, for the case of truncation at first or second order, the deviation is not negligible even if the error in measuring Ω_{GW} is taken into account. To present this clearly, we also plot the expected error in future direct detection experiments in Fig. 5. We assume that future experiments determine the value of Ω_{GW} with an accuracy of [70–72]

$$\sigma_{\Omega_{\text{GW}}} = 8.0 \times 10^{-18} \left(\frac{10^{-16}}{\Omega_{\text{GW}}} \right), \quad (32)$$

which is derived from the sensitivity of the BBO experiment (Detailed values for computing the noise spectrum are given in Ref. [69]). The Fabry-Perot type DECIGO has a similar sensitivity. Thus, the region within the error band would be similar to parameter space allowed by constraints from direct detection by DECIGO or BBO. Therefore, the use of power series expression of the spectrum may lead to incorrect parameter constraints from direct detection experiments, when one truncates it at lower order. However, if we includes up to the sixth order term, the estimate almost coincides with the true value.

B. ϕ^4 model with the curvaton

Next, we show an example of parameter estimation for the quartic potential in the presence of the contribution from the curvaton fluctuations to the primordial scalar perturba-

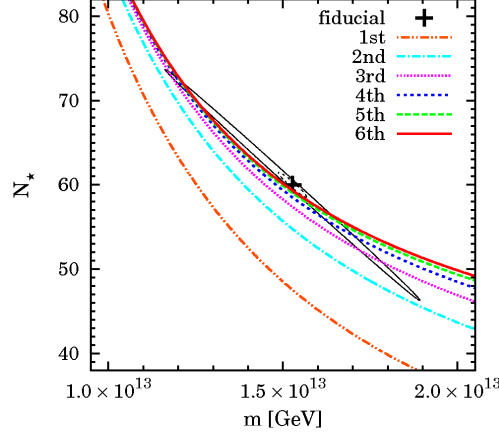


FIG. 4: Parameter estimation for the ϕ^2 model. The values of m and N_* are inferred from direct detection of the inflationary gravitational wave background with $\Omega_{\text{GW},0.2\text{Hz}} = 1.54 \times 10^{-16}$. Each line represents the values derived assuming the gravitational wave spectrum is described by Eq. (29), truncated at first, second, third, fourth, fifth and sixth order, respectively. The fiducial point is shown as a cross mark. The ellipses are the marginalized 2σ constraints expected from Planck (solid) and CMBpol (dashed).

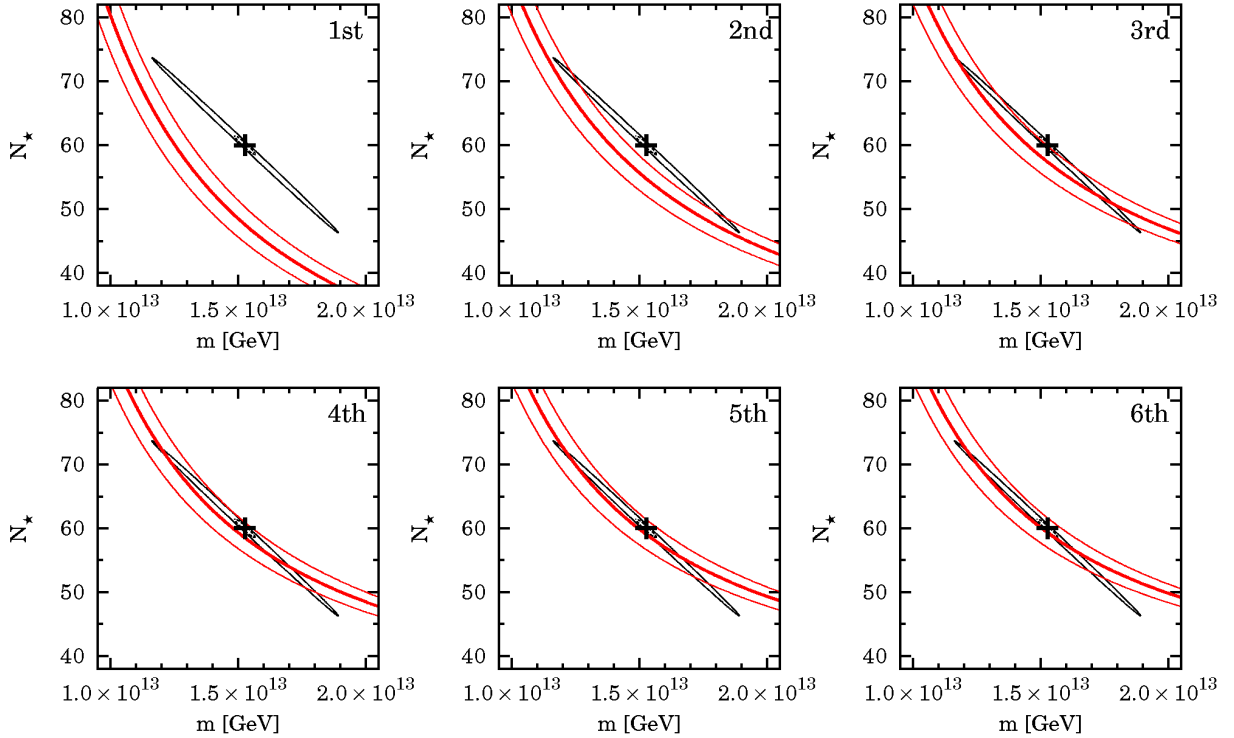


FIG. 5: The values of m and N_* inferred from the determination of $\Omega_{\text{GW},0.2\text{Hz}}$ with the 2σ experimental error of DECIGO/BBO. Each panel is for a different order truncation.

tions. In the same way as in Sec. IIIB, we assume the tensor-to-scalar ratio to be $r = 0.1$. In this case, the amplitude of the gravitational wave background would be determined to be $\Omega_{\text{GW},0.2\text{Hz}} = 4.15 \times 10^{-17}$ by direct detection experiments. The determination of $\Omega_{\text{GW},0.2\text{Hz}}$ provides a relation between $\mathcal{P}_{T\star}$ and $\epsilon_{V\star}$ via

$$\Omega_{\text{GW},0.2\text{Hz}} = 1.36 \times 10^{-6} \mathcal{P}_{T\star} \exp[-2(38.7\epsilon_{V\star}) - \frac{2}{2!}(38.7\epsilon_{V\star})^2 - \frac{6}{3!}(38.7\epsilon_{V\star})^3 - \frac{12}{4!}(38.7\epsilon_{V\star})^4 - \frac{48}{5!}(38.7\epsilon_{V\star})^5 - \frac{240}{6!}(38.7\epsilon_{V\star})^6 + \dots], \quad (33)$$

where we have used Eqs. (8), (10), (14), (21) and $\ln(k_{0.2\text{Hz}}/k_\star) = 38.7$. It can be converted to the information on N_\star and λ by

$$N_\star = \frac{1}{\epsilon_{V\star}} - 1, \quad (34)$$

$$\lambda = \frac{3\pi^2\epsilon_{V\star}^2}{32} \mathcal{P}_{T\star}, \quad (35)$$

which follows Eqs. (22) and (26).

In Fig. 6, the values of λ and N_\star obtained from the determination of $\Omega_{\text{GW},0.2\text{Hz}}$ are shown for different order truncation of Eq. (33). The lines in the $\lambda - N_\star$ plane are plotted by Eqs. (34) and (35) with parameters $\mathcal{P}_{T\star}$ and $\epsilon_{V\star}$ satisfying Eq. (33). For the same reason as described in the previous subsection, the truncation of the higher order terms gives an incorrect estimation of the values of λ and N_\star . The deviation from the true value is larger than the ϕ^2 case, because of the larger overestimation of the spectrum as presented in Sec. III. In this case, the overestimation may come not only from the truncation of the higher order terms of the power series expansion in terms of $\ln k$, but also those of the slow-roll approximation. In our numerical calculation, the slow-roll parameter is $\epsilon_V \simeq 4.56 \times 10^{-2}$ when the mode corresponding to 0.2 Hz exits the horizon during inflation. This means the second order slow-roll correction in $\mathcal{P}_{T,0.2\text{Hz}}$ (see Eq. (5)) can be a few percent around direct detection scales. Note that this cannot be improved even if we take into account the second order slow-roll correction as long as the spectrum is extrapolated from CMB scales, since the second order slow-roll correction is still small ($\epsilon_{V\star} \simeq 1.63 \times 10^{-2}$) when the modes corresponding to CMB scales exit the horizon.

In Fig. 7, the lines are shown with the experimental error of direct detection experiments, estimated by Eq. (32). The larger error than the ϕ^2 case is because of the smaller amplitude of the tensor spectrum due to the reduced normalization. Furthermore, since the CMB

constraints are obtained marginalizing over not only the cosmological parameters but also α_* characterizing the contribution of the curvaton fluctuations, the uncertainty becomes larger compared to the case for the ϕ^2 chaotic inflation model without the curvaton.

V. CONCLUSION

Inflation robustly predicts a stochastic gravitational wave background with a nearly scale-invariant spectrum. The detection of such gravitational waves is one of the next challenges in observational cosmology. If both CMB polarization and direct detection experiments achieve the detection, the independent information from the two different scales would provide a breakthrough in understanding the underlying physics of inflation.

Since the two different experiments measure gravitational waves at wavelengths separated by 16 orders of magnitude, the deviation from the scale-invariant spectrum, which is traditionally expressed by the power series expansion of $\ln k$, causes a large difference in amplitude of the primordial spectrum between two scales. The difference comes not only from the first order term of the power-law expansion, so-called the spectral index, but also from higher order terms, so-called runnings. We have presented that, in the case of chaotic inflation, the truncation of the running terms leads to the overestimation of the spectrum amplitude at the direct detection frequency. The overestimation is more prominent in the case where inflation predicts large slow-roll parameters. If we consider a single-field inflation model, large slow-roll parameters correspond to a large tensor-to-scalar ratio, in case of which we expect to detect the inflationary gravitational waves. Therefore, the overestimation of the tensor power spectrum should be carefully taken into consideration in case we achieve detection of the inflationary gravitational wave background.

Furthermore, we have investigated how the overestimation affects the determination of inflationary parameters. We have considered parameter constraints obtainable from future direct detection experiments, assuming a specific form of the inflation potential. We have presented two examples: a quadratic chaotic inflation model and mixed inflation and curvaton model with a quartic inflaton potential. In both cases, the use of truncated power-law spectrum causes an incorrect estimation of the inflationary parameters and it can be improved by adding higher order terms. For correct estimation of inflationary parameters, we need to take into account higher order terms, perform a numerical calculation, or develop a

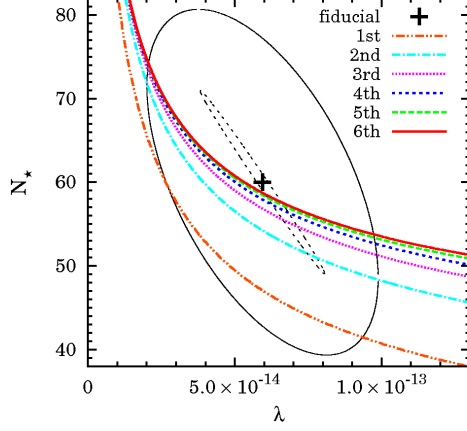


FIG. 6: Parameter estimation for the ϕ^4 model with the curvaton. The tensor-to-scalar ratio is assumed to be $r = 0.1$, which corresponds to direct detection with $\Omega_{\text{GW},0.2\text{Hz}} = 4.15 \times 10^{-17}$. Each line represents the values derived assuming the gravitational wave spectrum is described by Eq. (33), truncated at first, second, third, fourth and sixth order, respectively. The ellipses are the marginalized 2σ constraints expected from Planck (solid) and CMBpol (dashed).

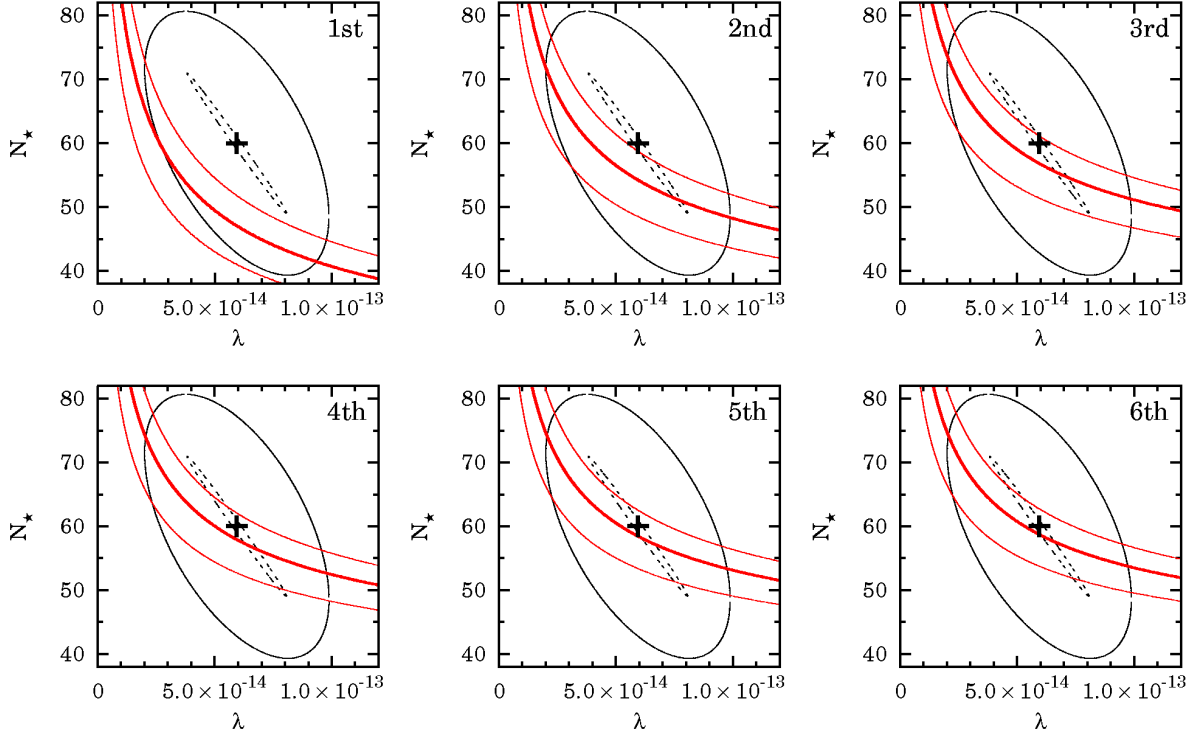


FIG. 7: The values of λ and N_* inferred from the determination of $\Omega_{\text{GW},0.2\text{Hz}}$ with the 2σ experimental error of DECIGO/BBO. Each panel is for a different order truncation.

new parametrization of the spectrum to connect the two separate scales.

Acknowledgments

The authors are grateful to Takeshi Chiba for helpful comments. SK would like to thank Toyokazu Sekiguchi, Takeshi Kobayashi and Shuichiro Yokoyama for discussions. TT would like to thank Kari Enqvist for discussions. The work of TT is partially supported by the Grant-in-Aid for Scientific research from the Ministry of Education, Science, Sports, and Culture, Japan, No. 23740195 and Saga University Dean's Grant 2011 For Promising Young Researchers.

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